OTS: 60-41104

JPRS: 5211

3 August 1960

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WASHINGTON 25, D. C.

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205 EAST 42nd STREET, SUITE 300
NEW YORK 17, N. Y.

JPRS: 5211
CSO: 4611-N

TOWARD THE CALCULATION OF THE INFLUENCE OF FRONTAL SEPARATIONS FOR THE SHORT RANGE PROGNOSES OF PRESSURE AND VERTICAL VELOCITIES

[This is a trnslation of an article by V.P. Sadokov, in Trudy, TSIP, No 6, 1957, pages 17-21; CSO: 4611-N]

At the present time, in the works concerning the short-range prognosis of meteorological elements by hydrodynamical methods, large-scale processes are studied. The usual consideration of such processes permits the use of the so called quasi-geostrophic condition for the analyses and ution of the hydrodynamical equations. It is considered, that in large-scale process the chief part of the wind velocity is determined by the geostrophic relation.

With the help of the quasi-geostrophic condition it is possible to write down the basic equations of dynamic meteorology (the vorticity equation and adiabatic equation) approximately in the form:

(1) $$\Delta \frac{\partial H}{\partial t} - \frac{\ell^2}{g} \frac{\partial \tau}{\partial p} = -\frac{g}{\ell}(H, \Delta H) - \frac{\partial \ell}{\partial y} \frac{\partial H}{\partial x}$$

(2) $$p \frac{\partial}{\partial p}\left(\frac{\partial H}{\partial t}\right) + \frac{m^2 \ell^2}{g} \cdot \frac{\tau}{p} = -\frac{g}{\ell} p \left(\frac{\partial H}{\partial p}, H\right)$$

Here the x, y, p, t system of coordinates is taken:

H height of an isobaric surface $p =$ constant,

$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

$\ell = 2\omega \sin \phi$ — the Coriolis parameter

ω angular velocity of the earth's rotation

ϕ geographical latitude

g — acceleration of gravity

$$m^2 = \frac{R^2 T(\gamma_a - \gamma)}{g l^2}$$

R — gas constant

T absolute temperature

γ_a, γ — corresponding adiabatic and ordinary temperature gradient

$$\tau = \rho g \left(\frac{\partial H}{\partial t} + u \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y} - w \right)$$

W — vertical velocity in the x, y, z, t coordinate system

ρ air density

u, v — velocity components

$$(A, B) = \frac{\partial A}{\partial x}\frac{\partial B}{\partial y} - \frac{\partial A}{\partial y}\frac{\partial B}{\partial x}$$

The system of equations (1)-(2) themselves present a non-linear system of differential equations for the unknowns H and τ. Their solution could be found numerically with the help of the fast-operating numerical calculating machines. In general, system (1)-(2) has an analytical solution for the functions $\frac{\partial H}{\partial t}$ and τ under certain limitations.

The problem concerning the calculation of frontal separations in the ...t range prognoses is set down by the following method. The atmosphere is considered as a two layered fluid, each layer of which corresponds to a warm or cold air mass. In each layer the parameter m is considered as constant, but different from one another.

At the boundaries let us use:

(3) 1) $p = 0, \quad \frac{\partial H}{\partial t} < \infty$

(4) 2) $p = P, \; w = 0$ (the earth is considered as flat), $P \approx 1000 \text{ mb}$

3) AT THE SURFACE OF SEPARATION

(5) a) $\frac{dp_1}{dt} = \frac{dp_2}{dt}$

(6) b) $V_{n_1} = V_{n_2}$

where V_n is the normal component of the velocity of displacement of the front. Index 1 corresponds to the warm air, index 2 - cold.

Conditions (5) and (6) make it possible, with the help of equation (2), to write down the condition for the function $\partial H/\partial t$ at the surface of separation. Further, equations (1) and (2) could be solved relative to $\partial H/\partial t$ under the condition, that the parameter m = constant. Since we are considering a two-layer atmosphere with a constant parameter in each layer, therefore it is possible to construct an equation for each layer, the analytical solution of which shall serve only for the given layer. With the help of the conditions, which follow from (5) and (6), both solutions can be fitted together.

For the construction of the solution the assumption was made, which consists in that, the surface of separation is presented as a quasi-horizontal surface. As a result we arrive at the necessity of solving the two-layer problem with the supplementary conditions (5) and (6) at the boundary of the two mediums. It follows that we keep in mind for this, that the adopted assumption cannot be interpreted as fact by the exclusion of the frontal slope from consideration, nevertheless we shall consider completely the basic kinematical and dynamical pecularities of the front, using (5) and (6).

Here the present method of linearization is available.

The solution of the problem is sought by the Fourier method. Finally the solutions are presented in the form of space integrals:[1/]

(7)
$$\frac{\partial u_1}{\partial t} = \frac{1}{4\pi}\iiint G_1^{(1)}(p, \Pi, p', r)\, F(x', y', p')\, d\sigma + \frac{1}{2\pi}\iint G_2^{(1)}(p, \Pi, r)\left[(u_1 - u_2)\frac{\partial f}{\partial x} + (v_1 - v_2)\frac{\partial f}{\partial y}\right] dS$$

(8)
$$\frac{\partial u_2}{\partial t} = \frac{1}{4\pi}\iiint G_1^{(2)}(p, \Pi, p', r)\, F(x', y', p')\, d\sigma + \frac{1}{2\pi}\iint G_2^{(2)}(p, \Pi, r)\left[(u_1 - u_2)\frac{\partial f}{\partial x} + (v_1 - v_2)\frac{\partial f}{\partial y}\right] dS$$

where F is a known function, which depends on the characteristics of the thermobaric fields of the atmosphere, $p = \Pi$ — the height of the front, $z = f(x, y, t)$ — the equation of the frontal surface in the x, y, z, t system of coordinates,

$$r = \sqrt{(x - x')^2 + (y - y')^2}\ , \quad G_1^{(1)},\ G_1^{(2)},\ G_2^{(1)},\ G_2^{(2)}$$

are Green's functions of the problem considered. We shall not give the expressions for the Green's functions here, but we shall only note, that they could be calculated analytically or, by means of the numerical solution of the differential operator of the system (1)-(2).

[1/] The solution of the problem in such a form was first obtained by N.I. Buleeyev and G.I. Marchuk.

In particular, the Green's functions $G_1^{(1)}$ and $G_1^{(2)}$ were computed by the latter method, by Hinkelmann [2].

The second integrals of formulas (7) and (8) describe the chief effect, arising from the presence of the frontal division. It depends on the distribution of the discontinuities in the wind field at the frontal surface and from the slope of the front. Here, integration is carried out along the frontal surface.

Thus we considered, in a complex way, a model of the atmosphere, which includes in its detail, the frontal processes with the usual large scale processes. It is completely obvious, that the frontal processes appear to attribute to the processes a smaller order than the large scale effects. This assertion could be confirmed by empirical evidence.

On the strength of the above-stated, it is possible to consider separately the large scale processes, described by the first integrals in (7) and (8), and the frontal processes described by the second integrals.

Independently analyzing the contributions of the two integrals on the general change of pressure, it follows that we keep in mind, that the relation between the characteristic horizontal and time scale calls for a calculation of the second integral for smaller time intervals, than is usually done.

We calculated the correlation coefficient between the actual 3 hour pressure change and the values of the two integrals calculated for that same interval of time. The correlation coefficient was found equal to 0.6 according to the results of 9 cases (over 100 points). Hence it is possible to conclude, that the frontal terms in the formulas (7,8) quite satisfactorily describes the real process in this time interval. The effect of the large-scale process in this case appears as a background, on which the effect of local,

process (in particular frontal processes) is superimposed.

Let us turn to consideration of the vertical motions. We shall be interested in those vertical motions, which arise in the region of the front and connected on the whole with the discontinuities in the wind field. Let us note, that contemporary methods of calculation of the vertical motions [1], [3] have in mind those motions, which arise as a result of large scale exchange between the kinetic, potential and intenal energies of the air mass in the atmosphere. It is possible to conisder the vertical motions of such a generation as a background. They can not describe those details in the distribution of vertical motions, which are observed, for example, at a front by virtue of the pecularities of its structure. In the future, we shall only consider those vertical motions, which are caused by the presence of the frontal separation, not dwelling on consideration of the large scale vertical motions.

Let us determine the vertical velocity from equation (1), integratin it with respect to the vertical and requiring that the quantity $\tilde{\tau}$ becomes zero at the upper and lower boundary of the atmosphere. Not paying attention to the large scale motions, described in equation (1) by the terms, contained on the right side in expressions (7,8) by the first integrals, it is possible to write

(9) $$\tilde{\tau}_1^* = \frac{g}{\ell^2}\int_0^p \Delta \frac{\partial H_1^*}{\partial t}\, dp$$

(10) $$\tilde{\tau}_2^* = \frac{g}{\ell^2}\int_P^p \Delta \frac{\partial H_2^*}{\partial t}\, dp$$

where

$$(11)\quad \frac{\partial H_1^*}{\partial t} = \frac{1}{2\pi}\iint G_2^{(1)}(p, \Pi, r)\left[(u_1-u_2)\frac{\partial \xi}{\partial x} + (v_1-v_2)\frac{\partial \xi}{\partial y}\right] dS$$

$$(12)\quad \frac{\partial H_2^*}{\partial t} = \frac{1}{2\pi}\iint G_2^{(2)}(p, \Pi, r)\left[(u_1-u_2)\frac{\partial \xi}{\partial x} + (v_1-v_2)\frac{\partial \xi}{\partial y}\right] dS$$

In expressions (11, 12), only the Green's functions alone depend on the variables p and x, y for that reason the integration with respect to p and differentiation $\left(\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$ must be carried out only over the Green's functions. As a result formulas (9,10), after substitution of expressions (11,12) into them, can be rewritten in the form:

$$(13)\quad \tilde{\tau}_1^*(\bar{p}, \Pi) = \frac{g}{2\pi l^2}\iint G_3^{(1)}(p, \Pi, r)\left[(u_1-u_2)\frac{\partial \xi}{\partial x} + (v_1-v_2)\frac{\partial \xi}{\partial y}\right] dS$$

$$(14)\quad \tilde{\tau}_2^*(p, \Pi) = \frac{g}{2\pi l^2}\iint G_3^{(2)}(p, \Pi, r)\left[(u_1-u_2)\frac{\partial \xi}{\partial x} + (v_1-v_2)\frac{\partial \xi}{\partial y}\right] dS$$

Here $G_3^{(1)}, G_3^{(2)}$ are new Green's functions. These functions could be tabulated under some assumptions $\left(\frac{m_1}{m_2} \approx 1\right)$

The expressions $(u_1-u_2)\frac{\partial \xi}{\partial x} + (v_1-v_2)\frac{\partial \xi}{\partial y}$ can be simplified, supposing, that $u \approx -\frac{g}{l}\frac{\partial H}{\partial y}$, $v \approx \frac{g}{l}\frac{\partial H}{\partial x}$.
From the condition at the front

$$(15)\quad p_1[x, y, \xi(x,y,t), t] \equiv p_2[x, y, \xi(x,y,t), t]$$

in particular, it follows, that

$$(16) \qquad \frac{\partial f}{\partial s} = \frac{\rho_2 \frac{\partial H_2}{\partial s} - \rho_1 \frac{\partial H_1}{\partial s}}{\rho_1 - \rho_2}$$

where $s = x, y, t$

Considering the above, we obtain

$$(17) \qquad (u_1 - u_2)\frac{\partial f}{\partial x} + (v_1 - v_2)\frac{\partial f}{\partial y} = \frac{g}{l}(H_1, H_2)$$

Thus, we manage to express the discontinuity in the wind at the front and its slope through the pecularities of the baric field at it.

In expressions (13,14), the Green's functions possess that characteristic, that they damp very rapidly from the forecast point. This means that the surroundings of the point of calculation of the field of the functions (H_1, H_2) have a weak influence on the result of the calculation. For that reason it is possible to simplify formulas (13,14), combining them into one by the following form

$$(18) \qquad \tau^{*}(p, \Pi) = a(p, \Pi)(H_2, H_1)$$

If we select for the unit of length in space as 500 km and we measure H in dkm (decameters), to determine all the meteorological parameters in the MTC system (meter-ten-second), then for the value $p =$ 0.8 (2km) the values of the coefficients a for the calculation of τ^{*} in mb/12 hours will be equal to: a (0.8; 1.0) = 0.16; a (0.8; 0.85) = 4.68;

a (0.8; 0.7) = 0.42; a (0.8; 0.5) = 0.28; a (0.8; 0.3) = 0.06.

The calculation of vertical motions in the frontal zone roughly comes to the following:

1. The location of the front is found on all the AT_{850}, AT_{700} and AT_{500} maps.

2. The positions of the front at all levels are placed on a separate clean form of a synoptic map [as if we project the fronts at all standard levels on to one plane (map)].

3. The necessary number of points are marked on the line of the front at each isobaric surface.

4. At the marked points the Jacobian $(H_2, H_1) =$

$$= \frac{\partial H_2}{\partial x} \cdot \frac{\partial H_1}{\partial y} - \frac{\partial H_2}{\partial y} \cdot \frac{\partial H_1}{\partial x}$$

is calculated. The calculations of the derivatives is carried out in finite differences. For this it is convenient to direct the x axis along the normal to the front on the side of the cold air, while the y axis is along the tangent (figure 1).

Assuming, that $\frac{\partial H_1}{\partial y} = \frac{\partial H_2}{\partial y}$, we obtain

(19) $$(H_2, H_1) = (H_2 - H_4)(H_1 - 2H_0 + H_3)$$

5. All the values (H_2, H_1), calculated at the points of the surface front were then multiplied by the coefficient a (1) = 0.16. The values (H_2, H_1) corresponding to the points of the front at the 850 mb surface, were multiplied by a (0.85) = 4.68 etc. These results were written down at the corresponding points on the clean blank (form), mentioned in (2), and then the map ~~ed~~ is drawn.

In operational work it is possible to use an accelerated method of calculation, which consists in the determination of the maximum values along the whole front. The actual or prognostic map AT_{850} is used for this purpose. The position of the front is found on it and (H_1, H_2) is calculated at the points of interest. Then the result is multiplied by the coefficient equal to 4.

In conclusion, we give the results of some examples of the calculation of τ^* on warm fronts. Calculations at cold fronts proved to be less successful, especially in the warm time of the year. The calculation of τ^* in mb/12 hr presented in figures 2, 3, 4, corresponds to the morning of March 24, 1953, the morning of September 14, 1953 and the evening of October 14, 1954 (the shaded regions are clouds at the time of the calcuhtions).

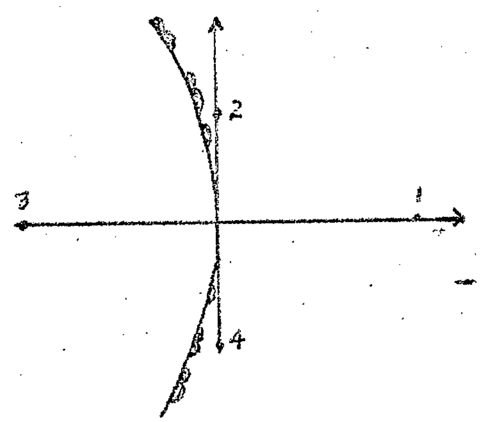

Fig 1

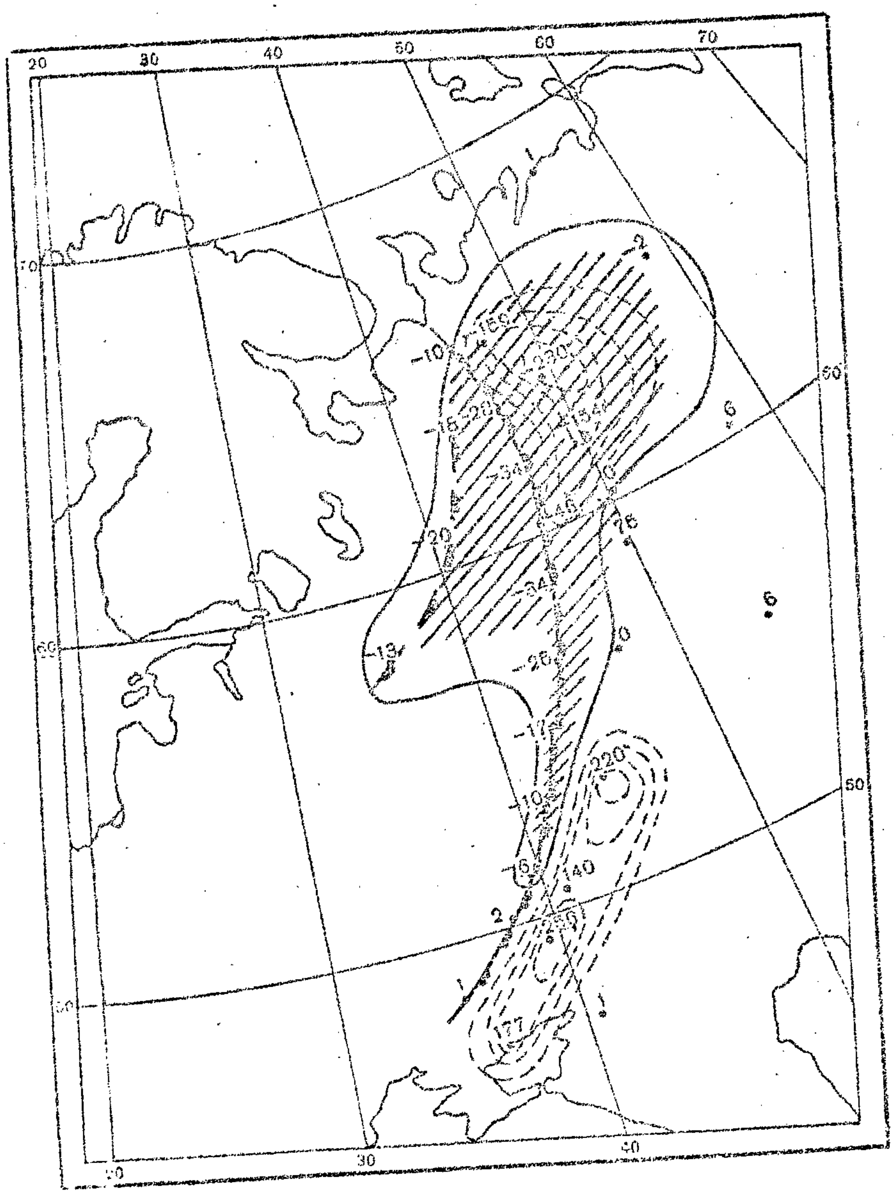

Рис. 2.
FIG

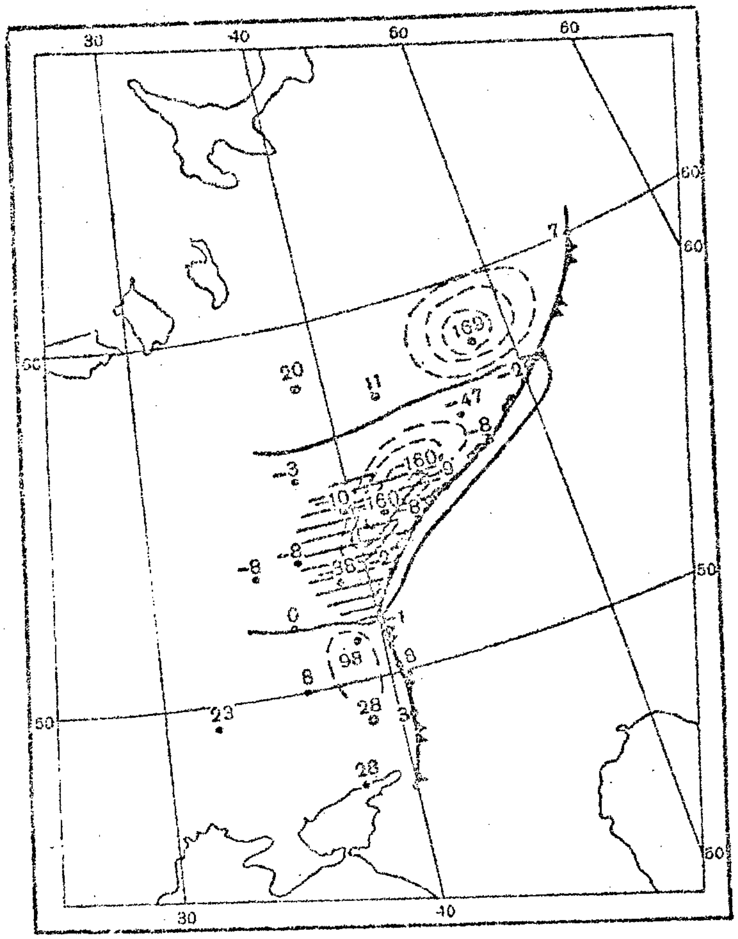

Рис. 3.
FIG

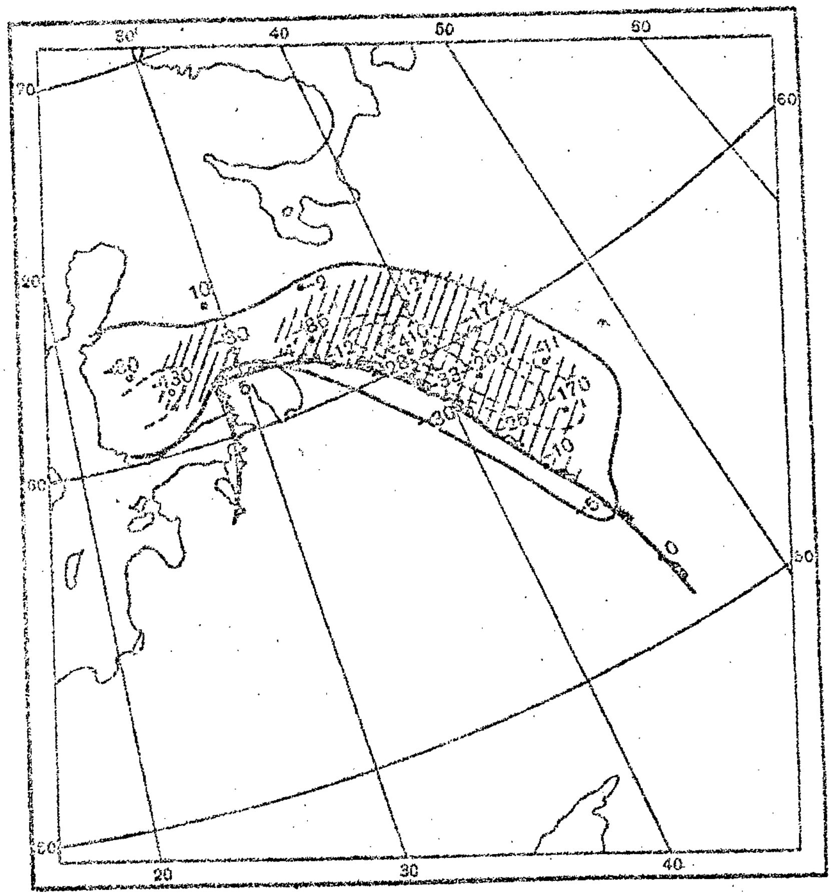

Рис. 4.
Fig.

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